Delay-Optimal Cross-Layer Design for Wireless Systems

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Outline

- Introduction and Motivation
- Survey of Existing Approaches
- Example:

  Distributive Delay-Optimal Control for Uplink OFDMA via Localized Stochastic Learning and Auction Game

  - Convergence Analysis
  - Asymptotic Optimality
- Conclusion

Introduction and Motivation

Why delay performance is important?

- "WHAT??!! He is stuck in the air!! !$*(&#%*!(!"
- "You must be kidding me! Buffering at such an important moment!!??"

You Tube

Fact I:
Real-life applications are **delay-sensitive**

Introduction and Motivations

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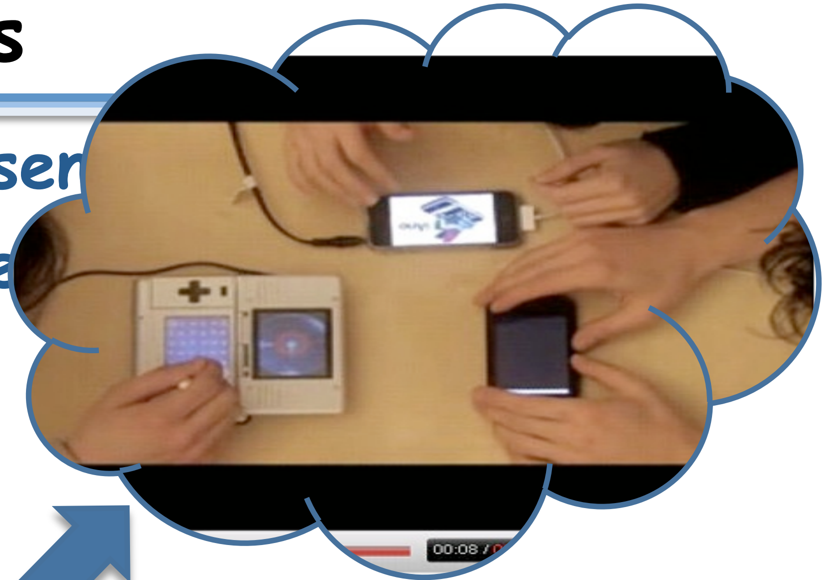
Keep track of a game



Play multi-player game

Fact II:
Different users and applications have **heterogeneous delay requirements**

Keep talking to some friends

Related Works

OFDMA Joint Power and Subband Design for PHY Performance

[Yu'02], [Hoo'04],[Seong'06], etc.

– Selects the strongest user per subband
– Time-Frequency Water-filling Power Allocation
– Assuming knowledge of **perfect CSIT**.

[Lau'05], [Wong'09], [Brah'07] etc.

– Robust Power and Subband Control with **limited feedback** or **outdated CSIT (packet errors)**.

Remark:
Only adapt based on **CSIT**, ignoring queue states and optimize **PHY layer performance only (throughput or PFS)**

Conclusion: **Very important to make use of both (channel state info) CSI and (queue state info) QSI for delay sensitive applications**

Introduction and Motivations

Challenges to incorporate QSI and CSI in adaptation

Challenge 1: Requires both **Information theory** (modeling of the PHY dynamics) & **Queueing theory** (modeling of the delay/buffer dynamics)

Challenge 2: Brute-force approach cannot lead to any viable solution

Information Theory + **Queueing Theory**

Claude Shannon

Leonard Kleinrock

When Shannon meets Kleinrock…

Existing Approaches to deal with Delay-Optimal Control

Various approaches dealing with delay problems

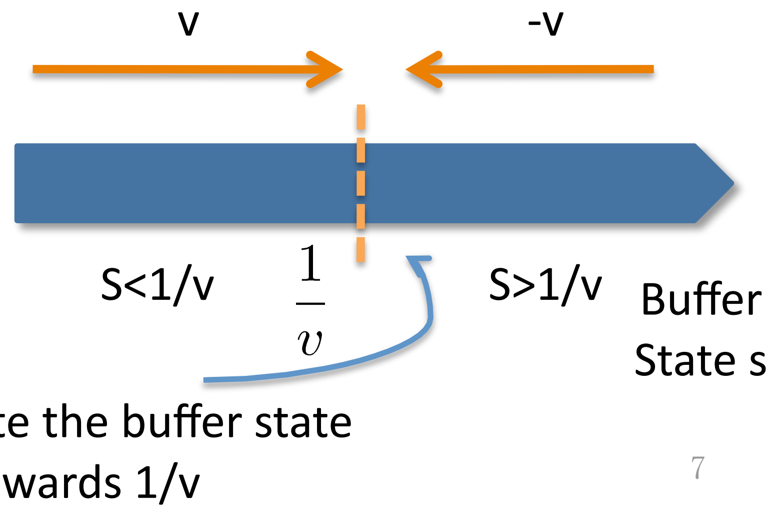
Approach I : Stability Region and Lyapunov Drift [Berry'02], [Neely'07], etc.

- Discuss **stability region** of point-to-point SISO and multiuser SISO.
- Also considered asymptotically delay-optimal control policy based on "**Lyapunov Drift**"
- The authors obtained interesting tradeoff results as well as insight into the structure of the **optimal control policy at large delay regime**.

Remark:

This approach allows simple control policy with design insights but the control will be good only for asymptotically **large delay regime**.

Buffer Partitioning



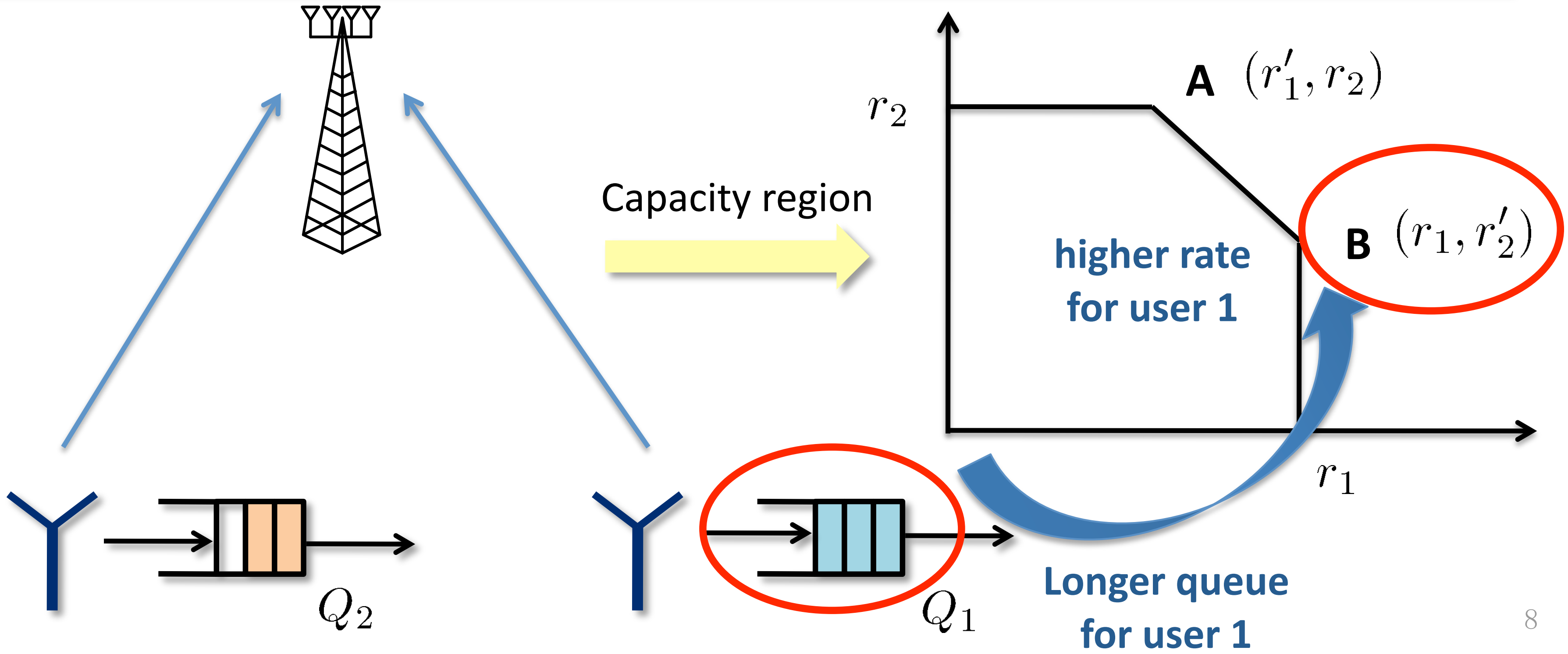
Related Works

- **Various approaches dealing with delay problems**

Approach II [Yeh'01PhD], [Yeh'03ISIT]

- Symmetric and homogeneous users in multi-access fading channels

- Using **stochastic majorization theory**, the authors showed that the **longest queue highest possible rate (LQHPR)** policy is delay-optimal



Related Works

Various approaches dealing with delay problems

Approach III : [Hui'07], [Tang'07], etc.

To **convert the delay constraint into average rate constraint** using tail probability at large delay regime (large derivation theory) and solve the optimization problem using information theoretical formulation based on the rate constraint.

Remark:

While this approach allows potentially simple solution, the control policy will be **a function of CSIT only** and such control will be good only for **large delay regime**.

Note:

In general, the delay-optimal power and precoder adaptation should be a function of both the CSI and the QSI.

Related Works

Various approaches dealing with delay problems

Approach IV : [Bertsekas'87]

The problem of finding the optimal control policy (to minimize delay) is casted into a **Markov Decision Problem (MDP)** or a stochastic control problem.

Remark:

– Unfortunately, it is well-known that there is no easy solution to MDPs in general.
– Brute-force **value iteration** and **policy iteration are very complex and time-consuming.**
– The curse of **dimensionality**!!

Technical Challenges To be Solved

Challenge 1:
A systematic approach for low complexity delay-optimal control policy in general delay regime.

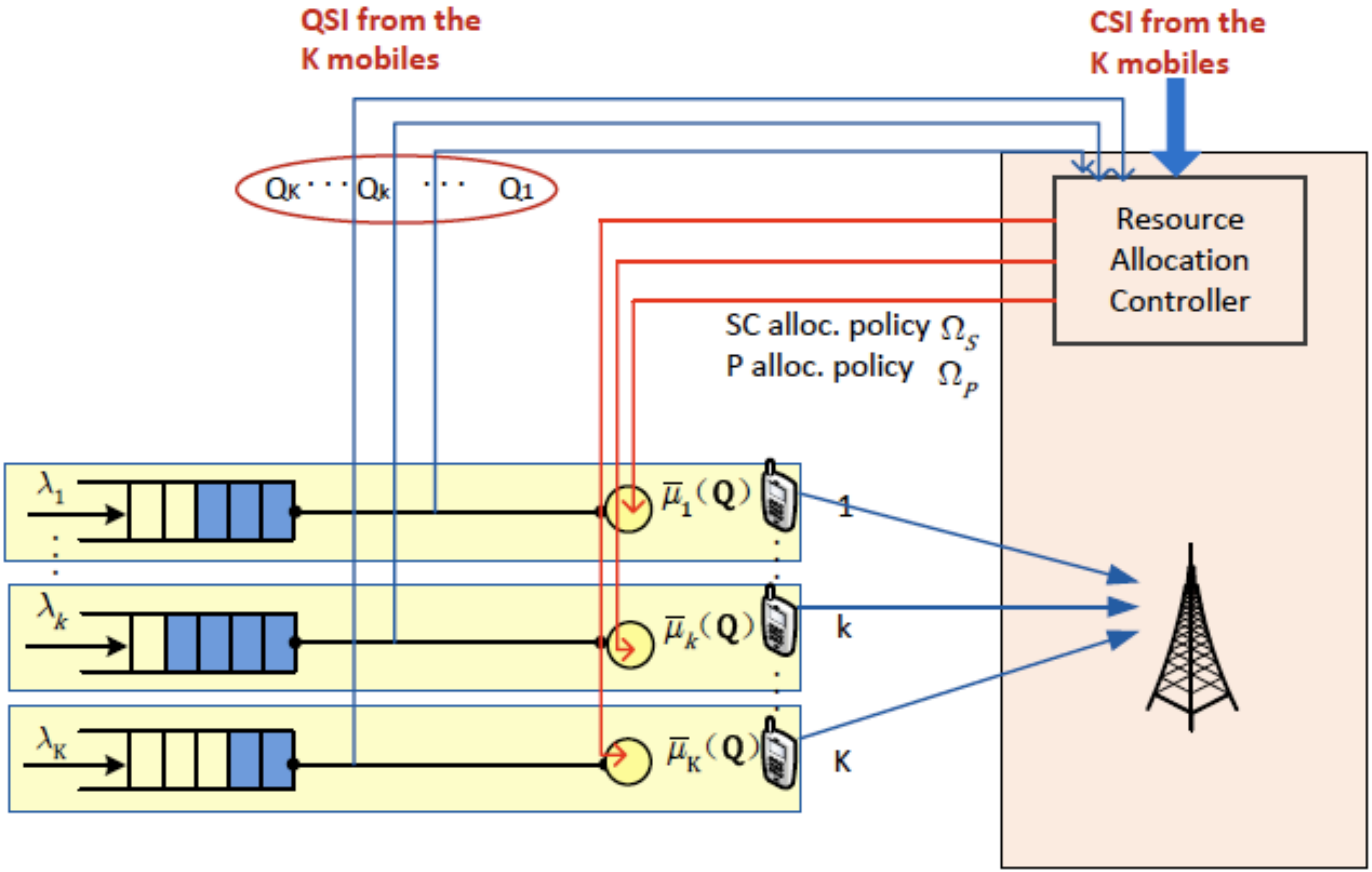
Challenge 2:
Curse of Dimensionality: Exponential Complexity due to coupling among multiple delay-sensitive heterogeneous users.

Challenge 3:
Structure of the delay-optimal policy.

Challenge 4:
Distributive Implementation (Function of Local CSI and Local QSI, e.g. uplink OFDMA)

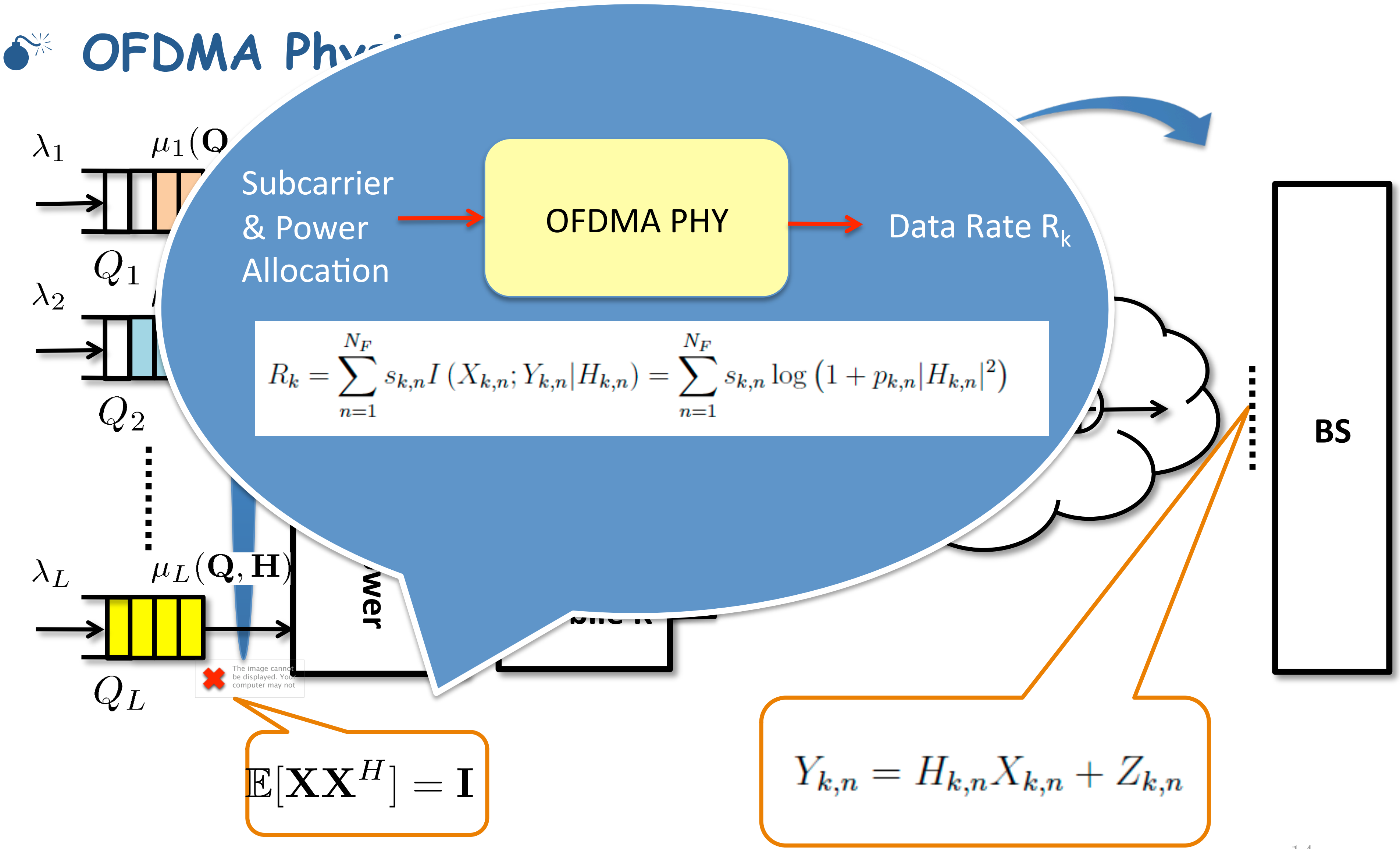
Example :
Distributive Delay-Optimal Control for Uplink OFDMA via Localized Stochastic Learning and Auction Game

Uplink OFDMA System Model

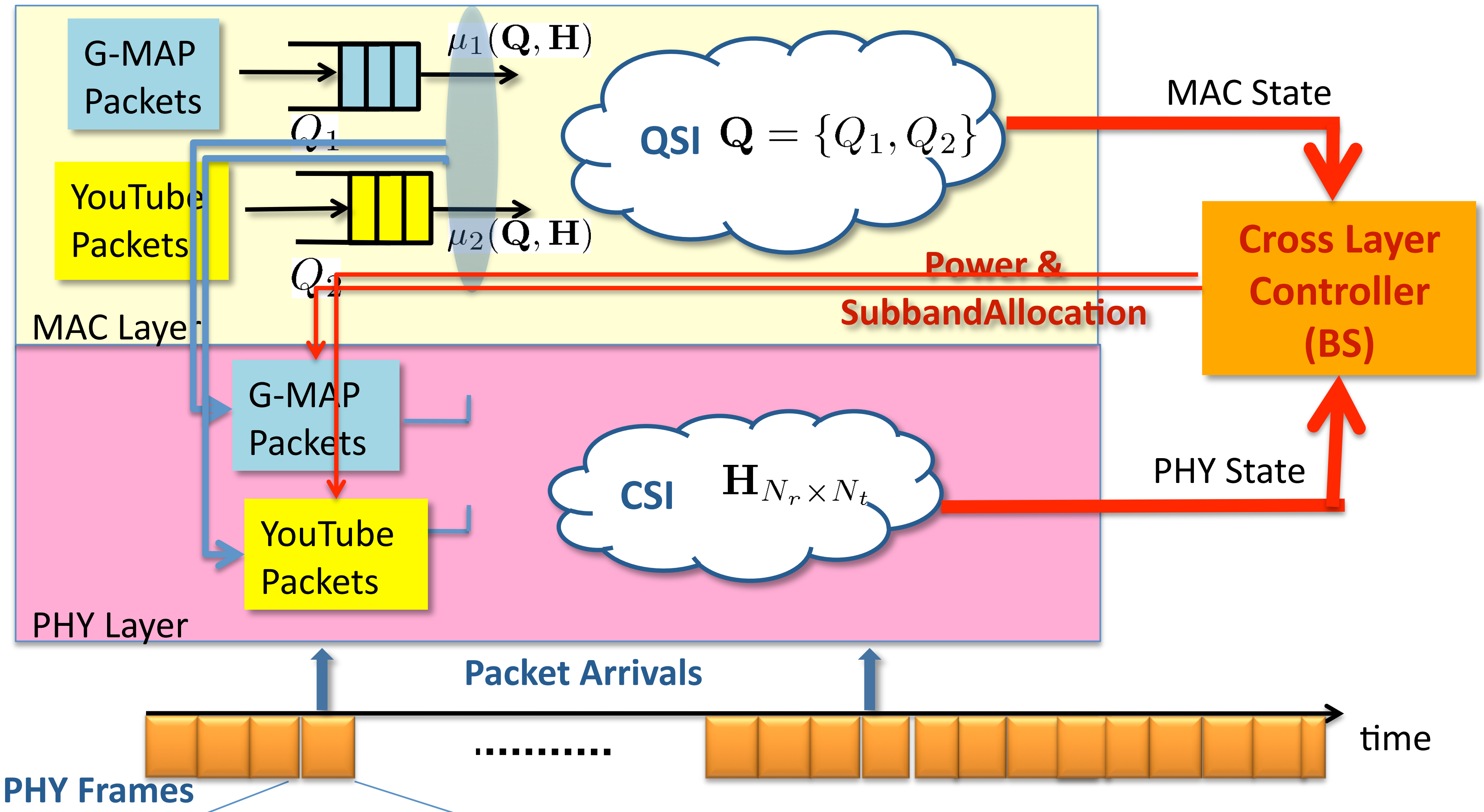


OFDMA PHY Model

- OFDMA Phy



Source Model and System States



scheduling time slot

- Channel is quasi-static in a slot
- i.i.d. between slots

OFDMA Queue Dynamics

- Time domain partitioned into scheduling slots
  - CSI H(t) remains quasi-static within a slot and is i.i.d. between slots
  - Packet arrival $A(t)=(A_1(t), \ldots, A_K(t))$ where $A_k(t)$ i.i.d. according to a general distribution P(A).
  - $N_k(t)$ denotes the random packet size, i.i.d.
  - $Q_k(t)$ denotes the number of packets waiting in the k-th buffer at the t-th slot.

$$Q_k(t+1) = \min\{[Q_k(t) - R_k(t)\tau/N_k(t)]^+ + A_k(t), N_Q\}$$

Total number of bits Transmitted in the t-th slot

  - Global System State (CSI, QSI)

$$\chi(t) = (\mathbf{H}(t), \mathbf{Q}(t))$$

OFDMA Delay-Optimal Formulation

Stationary Power and Subband Allocation Control Policy

- A mapping $\Omega = (\Omega_p, \Omega_s)$ from the system state χ to a power and subband allocation actions.

$$\Omega_p(\chi) = \{p_{k,n}\}, \quad \Omega_s(\chi) = \{s_{k,n}\}$$

$$\sum_{n=1}^{N_F} E\left[p_{k,n}\right] \le P_k \quad \forall k \in \{1, K\}, \quad p_{k,n} \ge 0$$ (Power Constraint)

$$\sum_{k=1}^{K} s_{k,n} = 1 \quad \forall n \in \{1, N_F\}$$ (Subband Allocation Constraint)

$$\Pr[Q_k = N_Q] \le P_k^d \quad \forall k \in \{1, K\}$$ (Packet Drop Rate Constraint)

OFDMA Delay-Optimal Formulation

Definitions: Average Delay, Power and Packet Drop Constraints under a control policy Ω

$$\overline{T}_k(\Omega) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[Q_k(t)] = \mathbb{E}_{\pi_\chi}[Q_k] \; \forall k \in \{1, K\}$$

$$\overline{P_k}(\Omega) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\Big[\sum_n p_{k,n}(t)\Big] = \mathbb{E}_{\pi_\chi}\Big[\sum_n p_{k,n}\Big] \le P_k \quad \forall k \in \{1, K\}$$

$$\overline{P_k^d}(\Omega) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\Big[\mathbf{1}[Q_k(t) = N_Q]\Big] = \mathbb{E}_{\pi_\chi}\Big[\mathbf{1}[Q_k = N_Q]\Big] \le P_k^d \quad \forall k \in \{1, K\}$$

$\mathbb{E}_{\pi_\chi}$ denotes expectation w.r.t. the underlying measure π.

Little's Law: average no. of packets=average arrival rate *average delay
the average delay (in terms of seconds) $\propto$ *the average queue length*

… Formulation

"Positive Weighting Factor"

$\beta = (\beta_1, \cdots, \beta_K)$

Pareto Optimal delay boundary

$d(\chi, \{\mathbf{p}, \mathbf{s}\}) = \sum_k \beta_k Q_k$

Pro… …imal control …

$$\min_{\Omega} J_{\beta}^{\Omega} = \sum_{k=1}^{K} \beta_k \overline{T}_k(\Omega) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[d(\chi(t), \Omega(\chi(t))) \right]$$

subject to the power and packet drop rate constraints

Solution: Markov Decision Problem (MDP)

Key Idea: Divide-and-Conquer

To break a large problem (optimization over the whole policy space) into smaller sub problems (optimization over a control action at a stage).

Overview of Markov Decision Problem Formulation

- **Specification of an Infinite Horizon Markov Decision Problem**
  - **Decisions are made at points of time – decision epochs**
  - **System state and Control Action Space**:
    - At the t-th decision epoch, the system occupies a state S_t
    - The controller observes the current state and applies an action A_t
  - **Per-stage Reward & Transition Probability**
    - By choosing action A_t the system receives a reward $R(S_t, A_t)$
    - The system state at the next epoch is determined by a transition probability kernel $\Pr(S_{t+1} \mid S_t, A_t)$
  - **Stationary Control Policy:**
    - The set of actions for all system state realizations $A_t = \pi(S_t)$
  - **The Optimization Problem:**
    - Average Reward
    - Optimal Policy

$$\overline{R}^* = \max_{\pi} \lim_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\sum_{t=1}^{T} R(S_t, A_t)\right]$$

Overview of Markov Decision Problem Formulation

- **Solution of an Markov Decision Problem**

> **Key Criterion: Bellman's Equation**
> Under some technical conditions, the optimal value of the problem is given by the solution of the **Bellman's Equation**.

- **Optimal average reward**

$$\overline{R}^* = \theta$$

- **Optimal policy (Fixed Point Problem on Functional Space)**

$$\pi^* = \arg\max_{A_i} \left\{ r(S_i, A_i) + \sum_{S_j} Pr(S_j | S_i, A_i) V(S_j) \right\}$$

Constrained Markov Decision Problem Formulation

$d(\chi, \{\mathrm{p}, \mathrm{s}\}) = \sum_k \beta_k Q_k$

CMDP Formulation: Find the optimal control policy Ω that minimizes

$$\min_{\Omega} J_{\beta}^{\Omega} = \sum_{k=1}^{K} \beta_k \overline{T}_k(\Omega) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\Big[d(\chi(t), \Omega(\chi(t)))\Big]$$

subject to the power and packet drop rate constraints

Lagrangian approach to the Constrained MDP:

$$\min_{\Omega} L_{\beta}^{\Omega}(\gamma) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\Big[g(\chi(t), \Omega(\chi(t)))\Big]$$

$$g(\gamma, \chi, \Omega(\chi)) = \sum_k \left(\beta_k Q_k + \overline{\gamma}^k (\sum_n p_{k,n} - P_k) + \underline{\gamma}^k (\mathbf{1}[Q_k = N_Q] - P_k^d)\right)$$

Optimal Solution

- Infinite Horizon Average Reward MDP
  - Given a stationary control policy Ω,
    $\{\chi(t), g(\chi(t), \Omega(\chi(t)))\}$ evolves like a Markov Chain
    with transition kernel:

$$\Pr[\chi(t+1)|\chi(t), \Omega(\chi(t))] = \Pr[\mathbf{H}(t+1)] \Pr[\mathbf{Q}(t+1)|\chi(t), \Omega(\chi(t))]$$

  - Solution is given by the "Bellman Equation"

$$\theta + V(\chi^i) = \min_{u(\chi^i)} \left[g(\chi^i, u(\chi^i)) + \sum_{\chi^j} \Pr[\chi^j|\chi^i, u(\chi^i)] V(\chi^j) \right]$$

"Potential function" (contribution of the state i to the average reward)

"Optimal Value" $\theta = J_\beta^* = \inf_\Omega J_\beta^\Omega$

$(N_Q+1)^K$ Equations and unknowns

Optimal Solution – Online Learning

- How to determine the potential function ?
  - Brute-Force solution of the Bellman Equation ? (Value Iteration):
    - Too complicated, exponential complexity and memory requirement
  - Online stochastic learning !
    - Iteratively estimate potential function based on real time observation of CSI and QSI – online value iteration
- Distributive Solution:

Per-user Potential and LMs Initialization

↓

Online Policy Improvement Based on Per-subband Auction

Online Per-user Potential and LMs Update [Local CSI, Local QSI]

↓

Termination

Decentralized Solution (I)

- Online Per-user Primal-Dual Potential Learning Algorithm via Stochastic Approximation

$$\widehat{V}^k_{l+1}(Q^i_k) = \begin{cases} \widehat{V}^k_l(Q^i_k) + \epsilon^v_l \Big[\big(g_k(\gamma^k_l, \chi_k(l)) + \widehat{V}^k_l(Q_k(l+1))\big) \\ \quad - \big(g_k(\gamma^k_l, \chi^I_k) + \widehat{V}^k_l(\bar{Q}^I_k) - \widehat{V}^k_l(Q^I_k)\big) - \widehat{V}^k_l(Q^i_k) \Big] & \text{if } Q^i_k = Q_k(l) \\ \widehat{V}^k_l(Q^i_k) & \text{if } Q^i_k \neq Q_k(l) \end{cases}$$

$$\overline{\gamma}^k_{l+1} = \Gamma\Big(\overline{\gamma}^k_l + \epsilon^\gamma_l \Big(\sum_n p^l_{k,n} - P_k\Big)\Big)$$

$$\underline{\gamma}^k_{l+1} = \Gamma\big(\underline{\gamma}^k_l + \epsilon^\gamma_l (\mathbf{1}[Q_k(l) = N_Q] - P^d_k)\big)$$

New Observation at the beginning of the (l+1)-th slot

$$\sum_l \epsilon^v_l = \infty, \epsilon^v_l \geq 0, \epsilon^v_l \to 0, \sum_l \epsilon^\gamma_l = \infty, \epsilon^\gamma_l \geq 0, \epsilon^\gamma_l \to 0$$

$$\sum_l \left((\epsilon^v_l)^2 + 2(\epsilon^\gamma_l)^2\right) < \infty, \frac{\epsilon^v_l}{\epsilon^\gamma_l} \to 0$$

Remark (Co...

Deterministic N...

CSI coherence tim...

...es evolves in the same

...etter solution (no

Both the per-user potential and 2 LMs are updated simultaneously.

Decentralized Solution (II)

- Per-stage auction with K bidders (MSs) and one auctioneer (BS)
  - Low complexity Scalarized Per-Subband Auction
    - Bidding: Each user submits a bid $\hat{X}_{k,n}$
    - Subband allocation: $s_{k,n}(\mathbf{H}_n, \mathbf{Q}^i) = \begin{cases} 1, & \text{if } k = k_n^* \text{ and } \hat{X}_{k_n^*,n} > 0 \\ 0, & \text{otherwise} \end{cases} \quad k_n^* = \arg\max_k \hat{X}_{k,n}$
    - Power allocation: $p_{k,n}(\mathbf{H}_n, \mathbf{Q}^i) = s_{k,n}(\mathbf{H}_n, \mathbf{Q}^i)\left(\frac{\frac{1}{N_k}\delta \widetilde{V}^k(Q_k^i)}{\gamma^k} - \frac{1}{|H_{k,n}|^2}\right)^+$
    - Charging: $C_{k,n} = s_{\bar{k}_n^*,n}\hat{X}_{\bar{k}_n^*,n}\mathbf{1}\{k = k_n^*\} \quad \bar{k}_n^* = \arg\max_{k \neq k_n^*} \hat{X}_{k,n}$

- Lemma: The per-stage social optimal scalarized bid

Water-level depends on QSI (via potential function)

$$\delta\widetilde{V}^k(Q_k^i) = \widetilde{V}(Q_1^i, \cdots, Q_k^i, \cdots, Q_K^i) - \widetilde{V}(Q_1^i, \cdots, [Q_k^i - 1]^+, \cdots, Q_K^i)$$

$\hat{X}_{k,n} \ldots - \frac{1}{|H_{k,n}|^2})^+$

Decentralized Solution

Theorem (Convergence of online per-user learning) Under some mild conditions, the distributive learning converges almost surely:

Remark (Comparison to conventional stochastic learning)

Conventional SL: (1) for unconstrained MDP only or LM for CMDP are determined offline by simulation; (2) designed for centralized solution with control action determined entirely from the potential update → Convergence Proof based on standard ***"contraction Mapping***" and ***Fixed-Point Theorem*** argument.

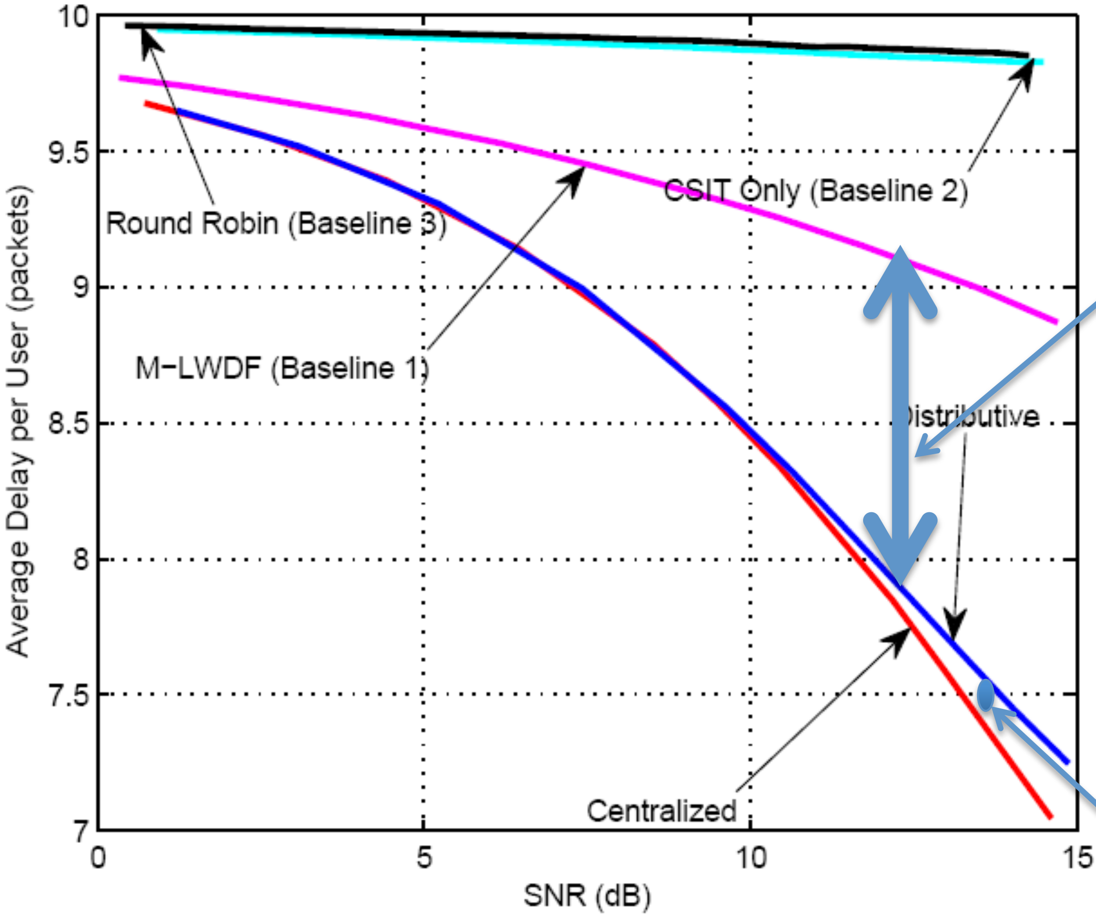
Proposed SL: (1) simultaneous update of LM and the potential function; (2) control action is determined by all the users' potential via per-stage auction → per-user potential update is ***NOT a contraction mapping*** & standard proof does not apply.

Bellman equation.

Numerical Results

Average Delay per user vs SNR

The number of users $K = 2$, the buffer size $N_Q = 10$, the mean packet size $\overline{N}_k = 305.2$ Kbyte/pck, the average arrival rate $\lambda_k = 20$ pck/s.



Huge gain in delay performance compared with Modified-Largest Weighted Delay First (M-LWDF) , which is the queue length weighted throughput maximization.

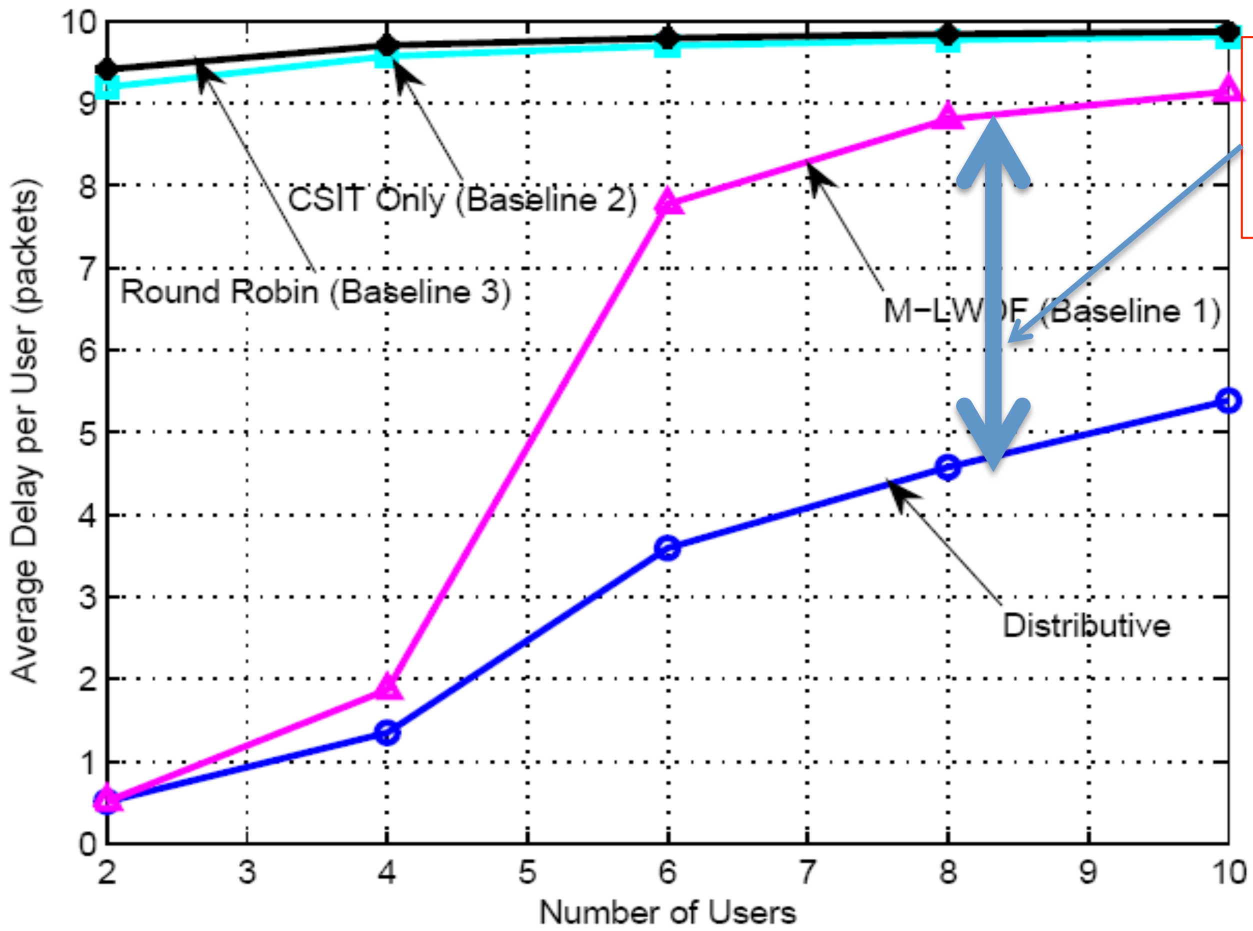
Close-to-optimal performance even for small number of users

Numerical Results

Average Delay per user vs No. of users

The buffer size $N_Q = 10$, the mean packet size $\overline{N}_k = 78.125$ Kbyte/pck

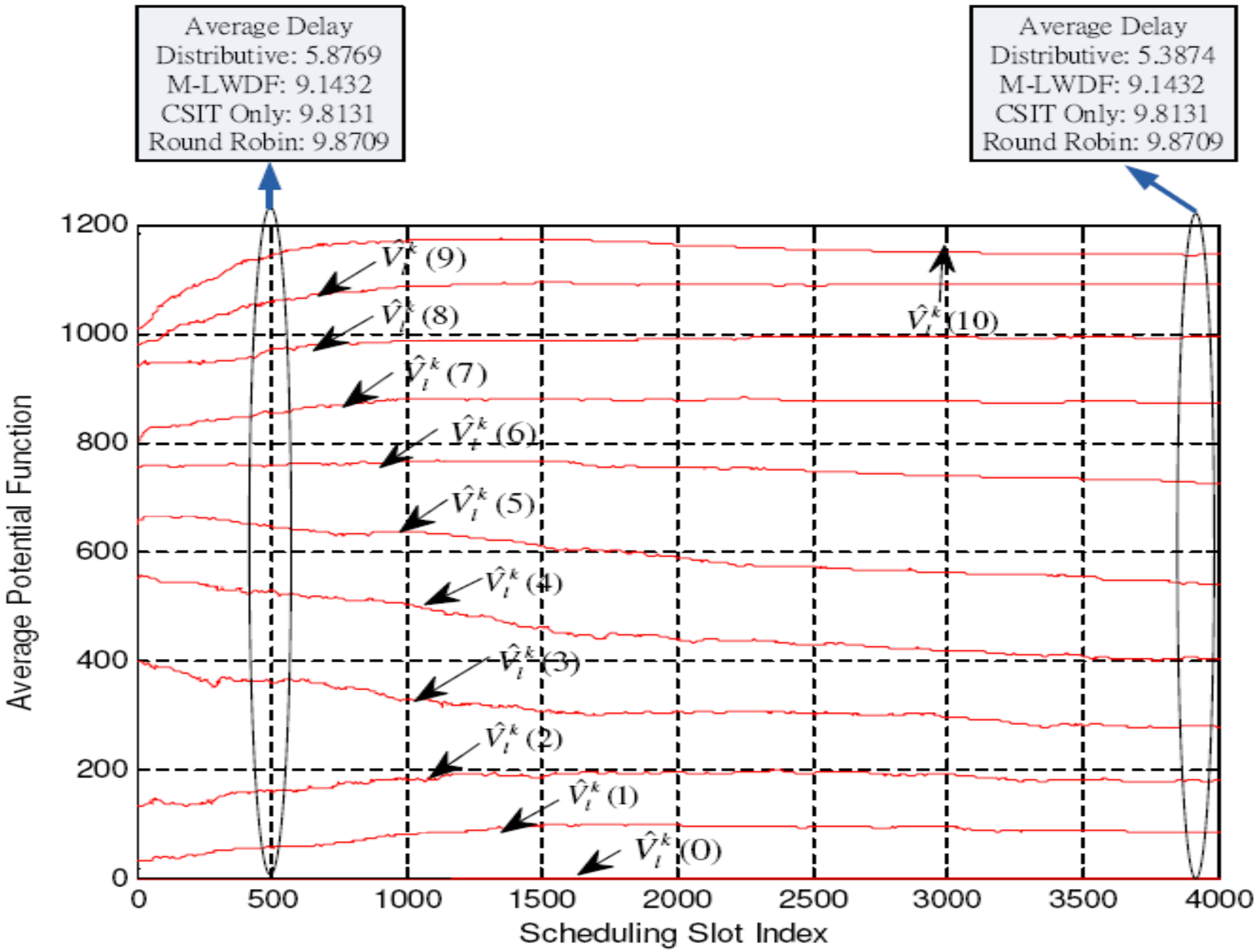
the average arrival rate $\lambda_k = 20$ pck/s, the queue weight $\beta_k = 1$ at a transmit SNR= 10dB.



The distributive solution has huge gain in delay performance compared with 3 Baselines.

Numerical Results

Illustration of convergence property: Potential function vs. the scheduling slot index (K=10)



Conclusion

Distributive Implementation via Decentralized Stochastic Learning and Auction Game

Online Per-user Learning:
Simultaneous update of LMs and Potentials.
Almost sure convergence

Optimal Strategy for the Auction Game:
Delay-Optimal Power Control: Multi-Level Water-Filling
(QSI→ water level; CSI→ instantaneous allocation)
Delay-Optimal Subband Allocation: User selection based on (QSI,CSI)

Asymptotically Global Optimal for large K

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Thank you!

Questions are Welcomed!

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