Multi-level Water-Filling for Delay-Optimal SDMA in Wireless Fading Channels

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Outline

- Introduction and Motivation
- System Model and Queue Dynamics
- Problem Formulation and Challenges
- Multi-Level Water-Filling Solution
- Asymptotic Analysis and Numerical Results
- Conclusion

Introduction

Why delay performance is important?

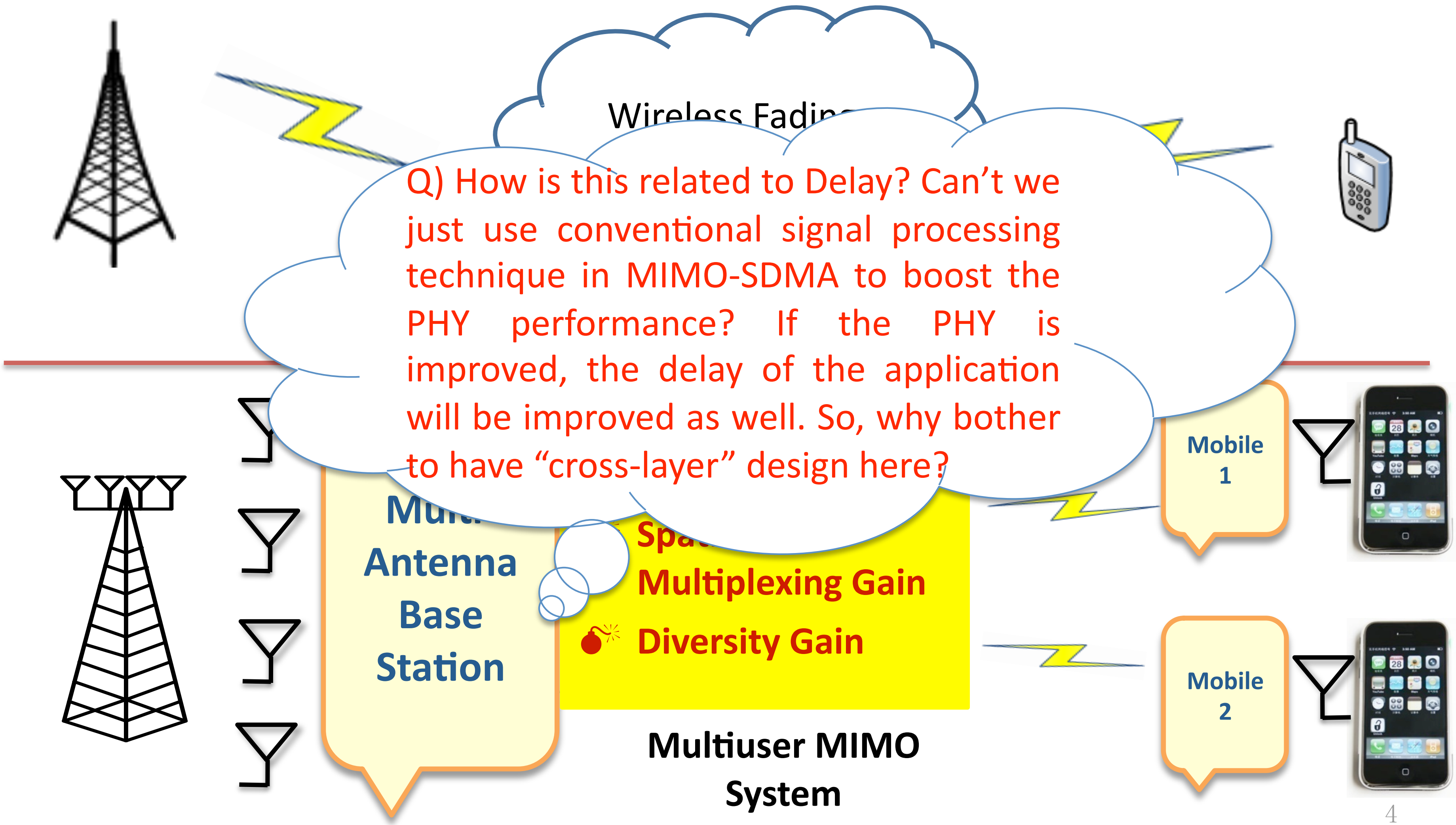
- "WHAT??!! He is stuck in the air!! !$*(&#%*!(!"
- "You must be kidding me! Buffering at such an important moment!!??"

You Tube

Conclusion I:
Real-life applications are **delay-sensitive**

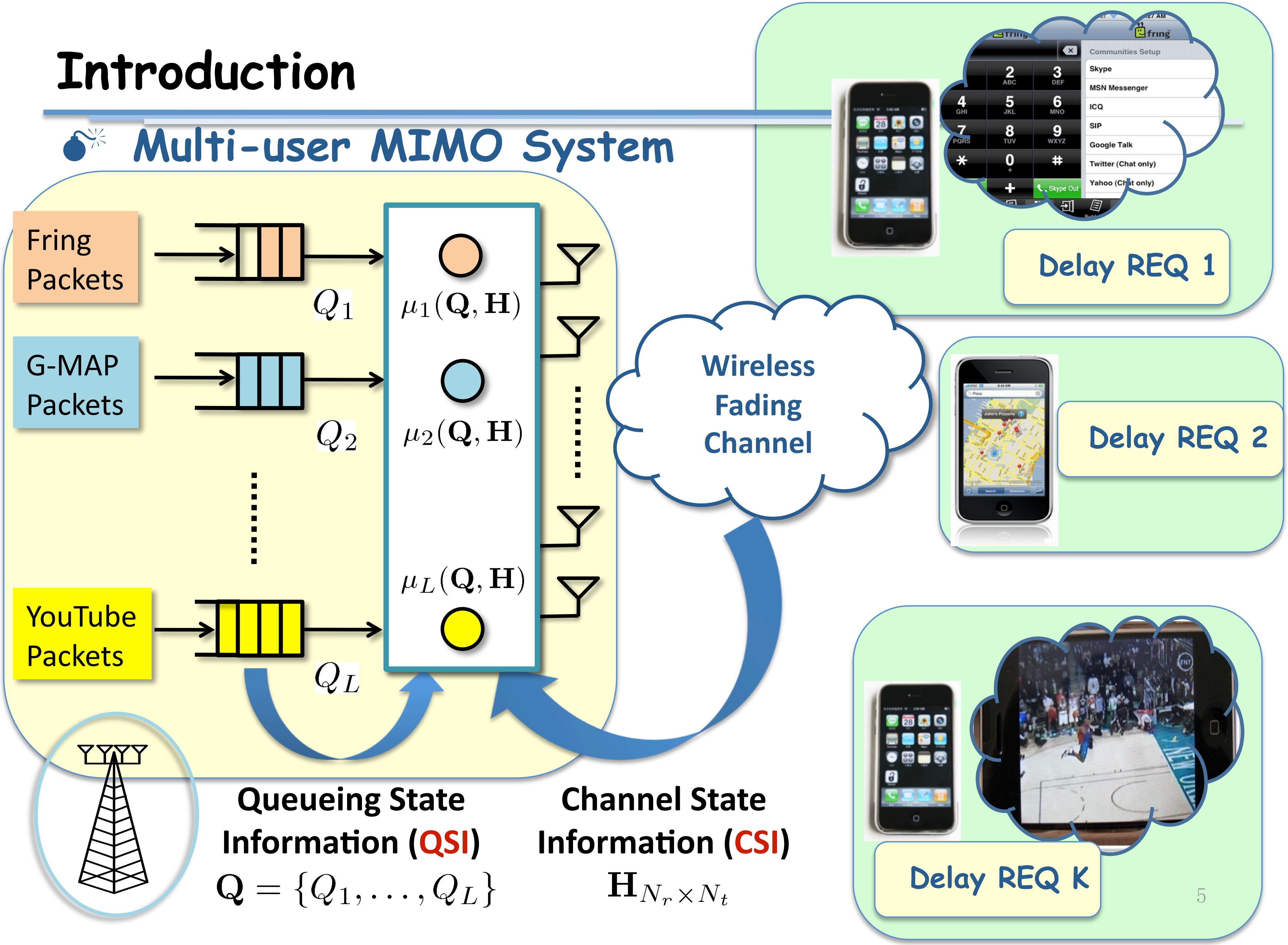
Introduction

- Multiuser MIMO - SDMA Systems to boost PHY performance



Introduction

Multi-user MIMO System



Related Works

SDMA Precoder Design for PHY Performance

[Sampath'01], [Scaglione'99],[Palomar'03], etc.

- Dirty Paper Coding (DPC) for MIMO Broadcast Channel
- Zero-Forcing Precoding for SDMA
- assuming knowledge of **perfect CSIT**.

[Love'05], [Lau'04], [Heath'04], etc.

- Precoder design for SDMA with **limited feedback**.
- Robust Precoder design for SDMA with **outdated CSIT**.

Remark:
Only adapt based on **CSIT**, ignoring queue states and optimize **PHY layer performance (throughput) only**

[Kittipiyakul'04] : naive water-filling, which is optimal in information theoretical sense, is not always a good strategy w.r.t. the **delay performance**.

Conclusion: **Very important to make use of both (channel state info) CSI and (queue state info) QSI for delay sensitive applications**

Introduction

- **Challenges to incorporate QSI and CSI in adaptation**

Challenge 1: Requires both **Information theory** (modeling of the PHY dynamics) & **Queueing theory** (modeling of the delay/buffer dynamics)

Challenge 2: Brute-force approach cannot lead to any viable solution

Information Theory + **Queueing Theory**

Claude Shannon

Leonard Kleinrock

When Shannon meets Kleinrock…

Related Works

Various approaches dealing with delay problems

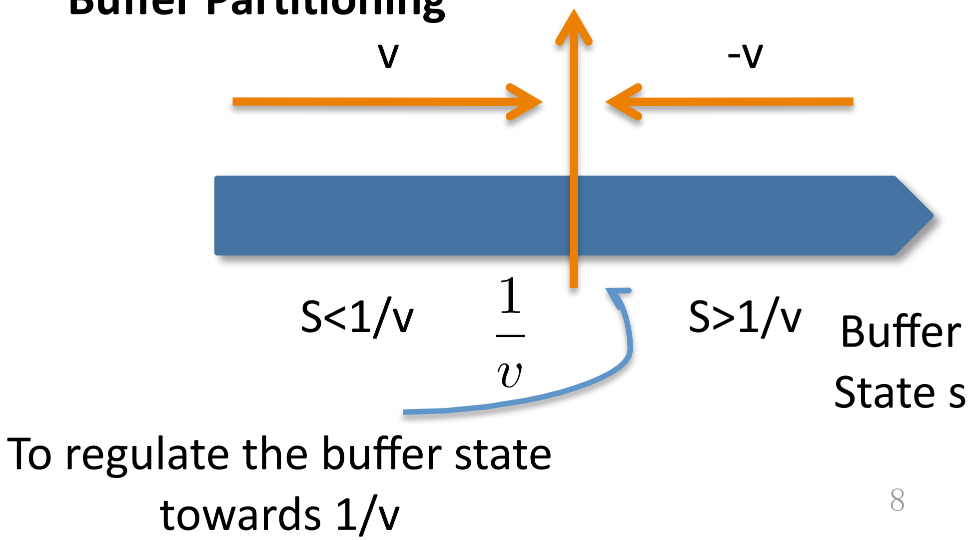
Approach I : Stability Region and Lynapnov Drift [Berry'02], [Neely'07], etc.

- Discuss **stability region** of point-to-point SISO and multiuser SISO.
- Also considered asymptotically delay-optimal control policy based on "**Lynapnov Drift**"
- The authors obtained interesting tradeoff results as well as insight into the structure of the **optimal control policy at large delay regime**.

Remark:

This approach allows simple control policy with design insights but the control will be good only for asymptotically **large delay regime**.

Buffer Partitioning



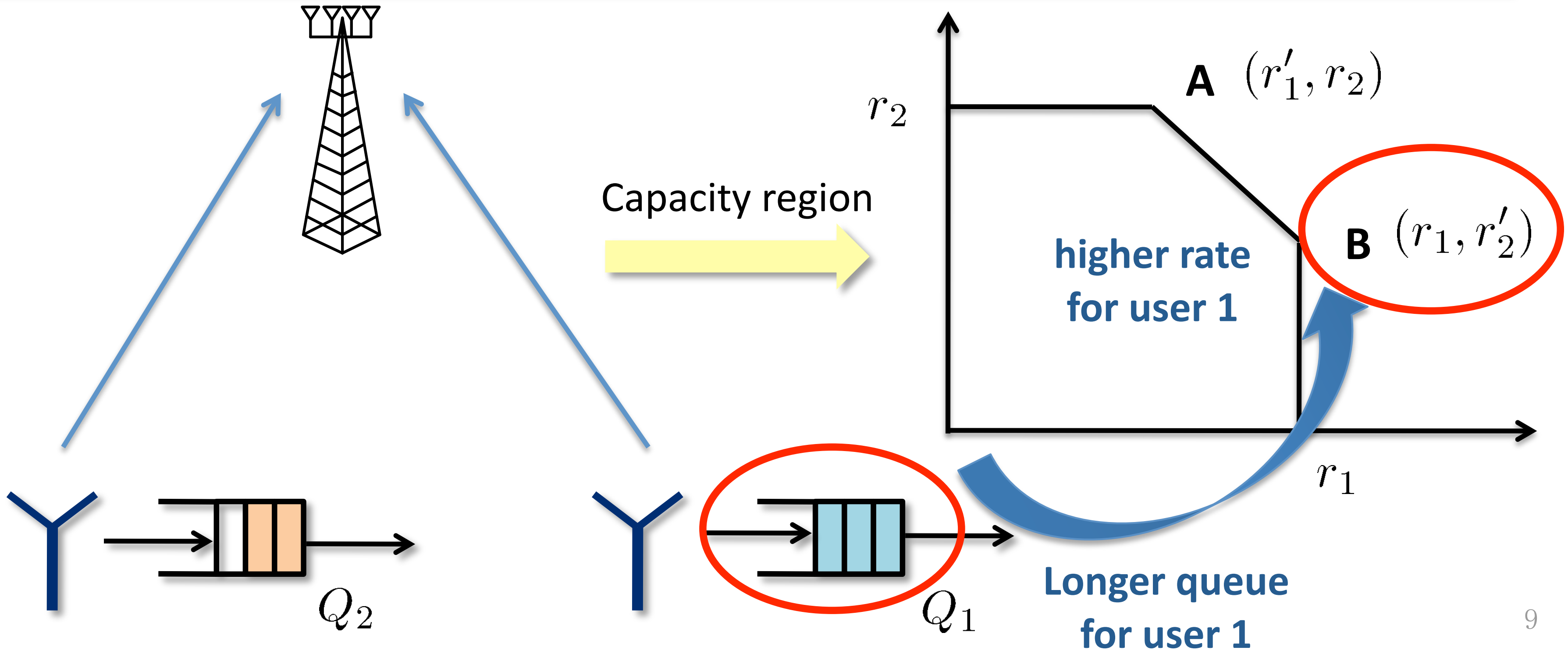
Related Works

- **Various approaches dealing with delay problems**

Approach II [Yeh'01PhD], [Yeh'03ISIT]

- Symmetric and homogeneous users in multi-access fading channels

- Using **stochastic majorization theory**, the authors showed that the **longest queue highest possible rate (LQHPR)** policy is delay-optimal



Related Works

Various approaches dealing with delay problems

Approach III : [Wu'03], [Hui'07], [Tang'07], etc.

To **convert the delay constraint into average rate constraint** using tail probability at large delay regime and solve the optimization problem using information theoretical formulation based on the rate constraint.

Remark:

While this approach allows potentially simple solution, the control policy will be **a function of CSIT only** and such control will be good only for **large delay regime**.

Note:

In general, the delay-optimal power and precoder adaptation should be a function of both the CSI and the QSI.

Related Works

Various approaches dealing with delay problems

Approach IV : [Bertsekas'87]

The problem of finding the optimal control policy (to minimize delay) is cast into a **Markov Decision Problem (MDP)** or a stochastic control problem.

Remark:

- Unfortunately, it is well-known that there is no easy solution to MDP in general.
- Brute-force **value iteration** and **policy iteration are very complex and time-consuming.**
- The curse of **dimensionality**!!

Related Works

Technical Challenges to be Solved

Challenge 1:
Low complexity delay-optimal control policy for delay sensitive SDMA systems in general delay regime.

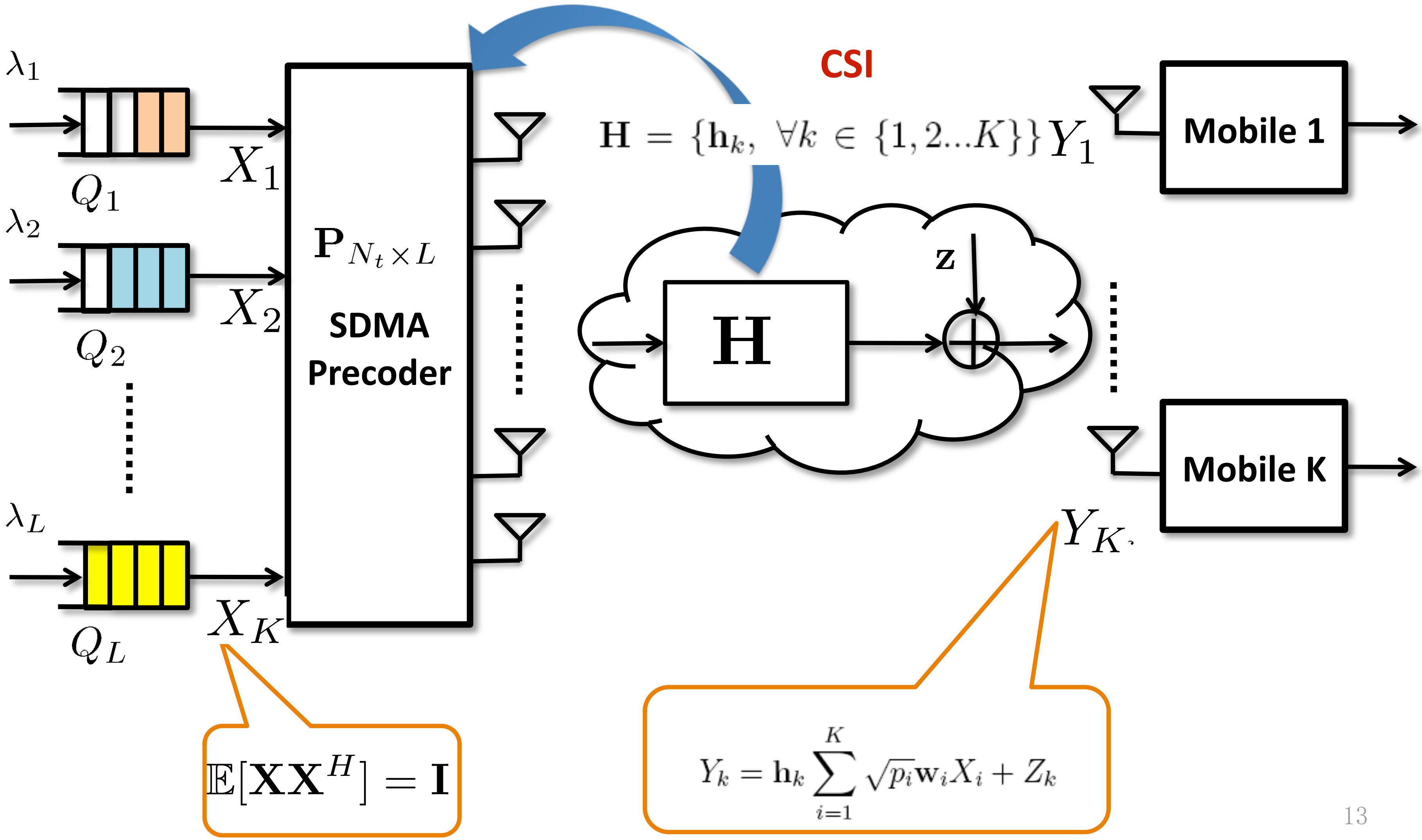
Challenge 2:
Exponential Complexity due to coupling among multiple delay-sensitive heterogeneous users.

Challenge 3:
Structure of the delay-optimal power control policy? How the power control depends on both the CSI and QSI?

Challenge 4:
Issue of Limited Buffer Size and Packet Dropping → How large the buffer has to be provisioned for a required packet drop rate?

System Model

Multiuser MIMO Physical Layer Model



System Model

SDMA Physical Layer Model

Equivalent channel for t

Zero-Forcing SDMA: $\mathbf{w}_k = A_k \left[\mathbf{I}_{N_t} - \mathbf{H}_k^*(\mathbf{H}_k^T\mathbf{H}_k^*)^{-1}\mathbf{H}_k^T\right]$

$$Y_k = \sqrt{p_k}\mathbf{h}_k\mathbf{w}_k X_k + Z_k$$

System St

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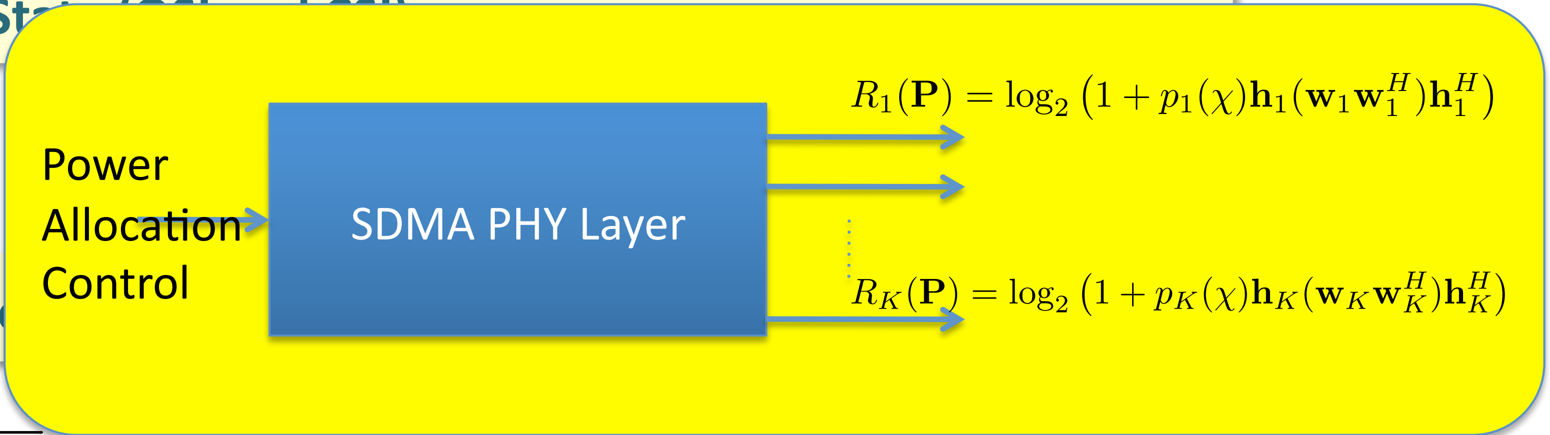


Power C

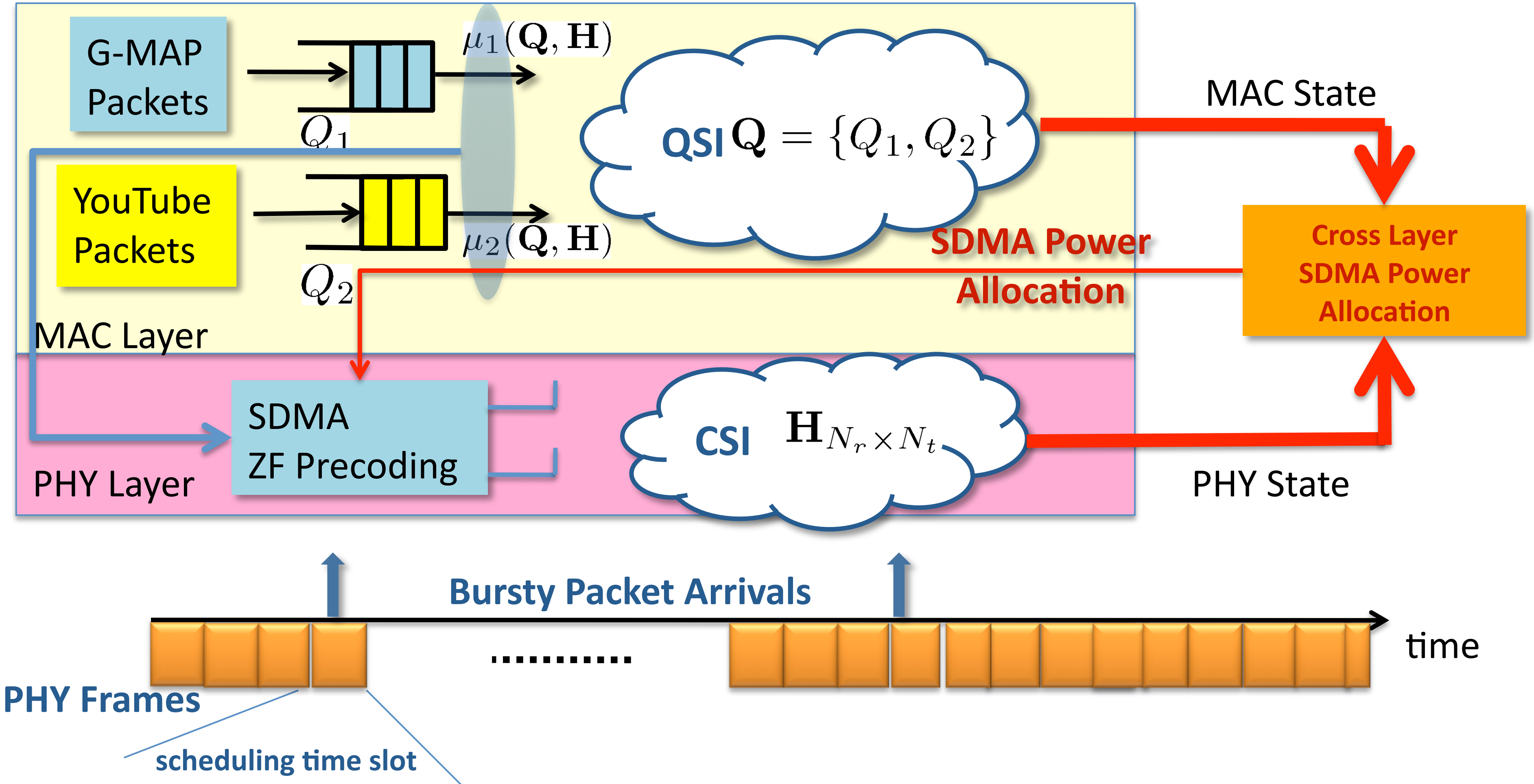
$$\mathcal{P} = \{(p_1(\chi), \ldots, p_K(\chi)) : \forall \chi\}$$

Data rate (bits per symbol) of the k-th user:

$$R_k(\mathbf{P}) = \log_2\left(1 + p_k(\chi)\mathbf{h}_k(\mathbf{w}_k\mathbf{w}_k^H)\mathbf{h}_k^H\right)$$

System Model

Queue Dynamics & System States



- Channel is quasi-static in a slot
- i.i.d between slots

System Model

O

System
Poisson
Average

Challenges:

- Huge dimension of variables involved (policy = set of actions over all system state realizations)
- K queues are coupled together → Exponentially Large State Space
- Problem not convex

Optimization problem: Delay Optimal Power Control Policy

$$\overline{T}^* = \min_{\mathcal{P}} \sum_{k=1}^{K} \frac{\boldsymbol{Q} \cdot \Pi_k(\mathcal{P})\tau}{\lambda_k}$$ (Total Average Delay of K users)

S.t.: $p_k(\chi) \geq 0$

$\pi_k(L) \leq \epsilon_d \quad \forall k \in \{1, 2...K\}$ (Packet Drop Rate Constraint)

$\sum_{k=1}^{K} \mathbb{E}_{\chi}[p_k(\chi)] = \sum_{k=1}^{K} \boldsymbol{P}_k \cdot \Pi_k(\mathcal{P}) \leq P_{avg}$ (Average Power Constraint)

System State Evolution

- Embedded Markov Chain
  - Sample the continuous time random process at frame boundaries $\{0, \tau, 2\tau, ...\}$, we have an "embedded discrete time random process": $\chi_m = (\mathbf{H}_m, \mathbf{Q}_m)$ where $\chi_m = \chi(m\tau)$

Lemma 1) For a given power control policy, the embedded random process $\chi_m = (\mathbf{H}_m, \mathbf{Q}_m)$ is a Controlled Markov chain with transition kernel given by:

$$\Pr[\mathbf{H}_{m+1}, \mathbf{Q}_{m+1} | \chi_m, \mathbf{p}(\chi_m)] = \Pi_{k=1}^{K} \Pr(\mathbf{h}_{k,m+1}) \Pr[\mathbf{Q}_{m+1} | \chi_m, \mathbf{p}(\chi_m)]$$

System State Evolution

- Sketch of Proof
  - Given the current state $\chi_m = (\mathbf{H}_m, \mathbf{Q}_m)$ and the control action $p_k(\chi_m)$, one of the following events could occur for user k at the (m+1)-th scheduling slot.

Packet arrival from the data source: Since packet arrival follows Poisson distribution with mean arrival rate λ_k, the transition probability of the buffer state corresponding to packet arrival is given by:

$$p_{k,q,q+1} = \Pr[Q_{k,m+1} = q+1 | Q_{k,m} = q] = 1 - e^{-\lambda_k \tau} \approx \lambda_k \tau \text{ for } q < L \quad (4)$$

Inter-packet arrival time >> t

Packet drop due to limited buffer size:

$$\eta_k = \frac{\Pr(\text{Packet arrival}|Q_{k,m} = L)\Pr[Q_{k,m} = L]}{\Pr(\text{Packet arrival})} = \frac{\lambda_k \tau \Pr[Q_{k,m} = L]}{\lambda_k \tau} = \Pr[Q_{k,m} = L] \quad (5)$$

Since the inter-arrival time of packets is memoryless, the above probabilities in (4) and (5) (conditioned on χ_m) is independent of the previous system states $\{\chi_{m-1}, \chi_{m-2}, \dots\}$.

System State Evolution

Sketch of Proof

Packet departure from the data buffer: A packet can depart if and only if the required service time of the remaining packet is no more than one slot duration. Since the packet length is exponentially distributed with mean packet length $\overline{N}_k$, the probability for packet departure at $t = (m+1)\tau$ (conditioned on the system state χ_m) is given by:

$$p_{k,q,q-1} = \Pr[Q_{k,m+1} = q-1 | Q_{k,m} = q, \chi_m, p_k(\chi_m)]$$

$$= \Pr\left(\frac{N}{\log_2(1 + p_k(\chi}\right.$$

$$\mu_k(\chi) = \frac{\log_2(1 + p_k(\chi)\mathbf{h}_k(\mathbf{w}_k\mathbf{w}_k^H)\mathbf{h}_k^H)}{\overline{N}_k}$$

$$= \Pr\left(\frac{N_k}{\overline{N}_k} < \mu_k(\chi_m)\tau\right) = 1 - e^{-\mu_k(\chi_m)\tau} \approx \mu_k(\chi_m)\tau \quad (6)$$

Mean Time to deliver a packet >> t

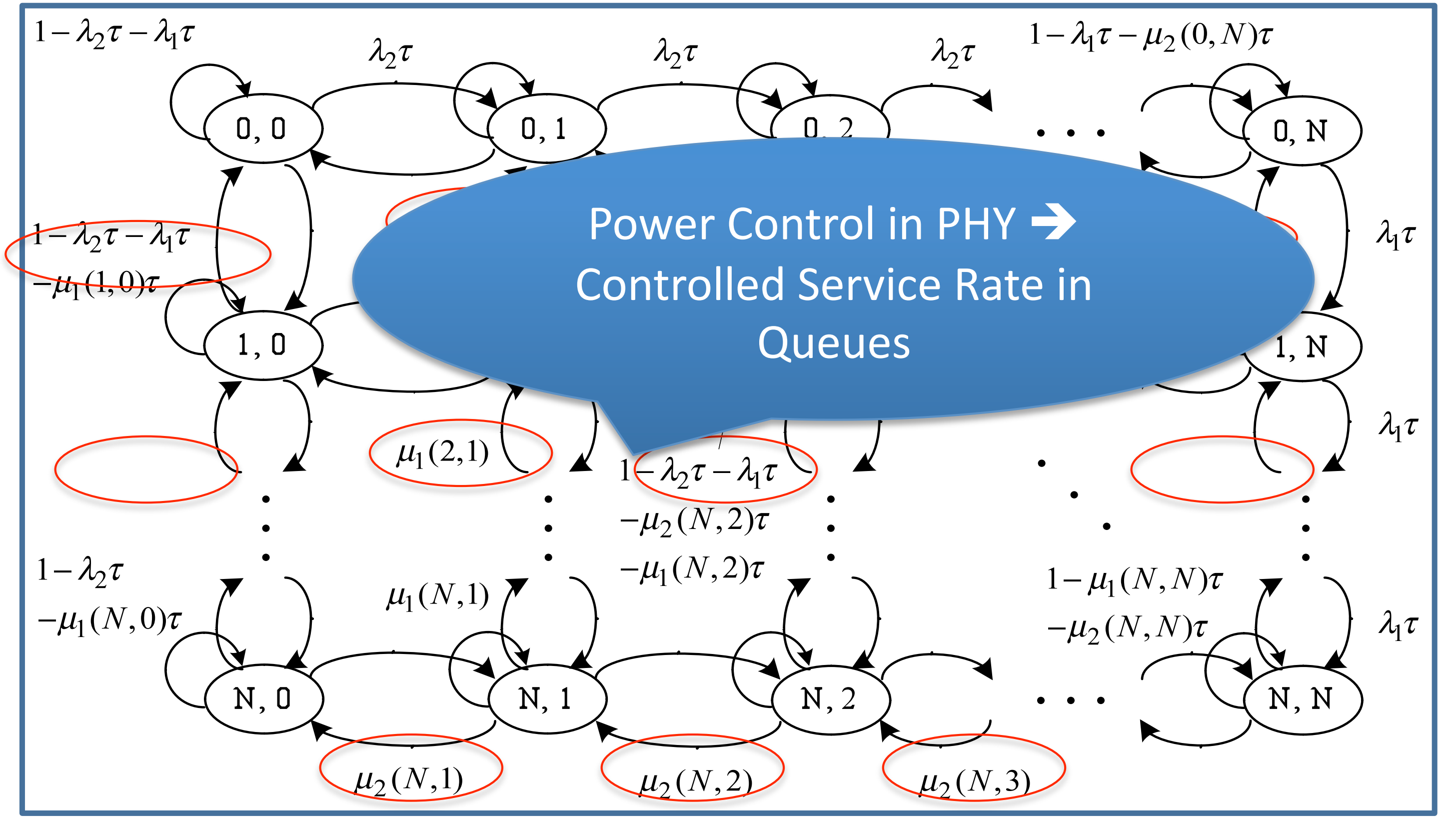
Since the packet length N_k is memoryless, the above probability (6) (conditioned on χ_m and action $p_k(\chi_m)$) is independent of the system state $\{\chi_{m-1}, \chi_{m-2}, \ldots\}$.

As a result of the memoryless property of the packet interrarrival and packet length distribution as well as (3), the embedded random process $\chi_m = (\mathbf{Q}_m, \mathbf{H}_m)$ is a discrete time Markov process. Furthermore, since $\lambda_k\tau$ and $\mu_k\tau$ are small, the probability of multiple packet arrivals or packet departures is of the order $\mathcal{O}[(\lambda_k\tau)^2]$ and hence is negligible.

System State Evolution

- **Our Transition Probability Kernel:**

State transition diagram for K-dimension Markov chain **{Qm}** with N states each dimension. K=2 for illustration.



Power Control in PHY → Controlled Service Rate in Queues

For unichain control policy, the induced Markov Chain is “aperiodic” and “irreducible”.

Technical Challenges

Major Challenges

1) Exponentially large Q state (QSI):

- The total number of states in the joint-queue-state (QSI) = N^L
- Exponentially large → complexity and memory requirement = O(exp[L])!!

2) Global Optimal Solution:

- The problem is not convex. How to make sure we have global optimal solution?

3) Asymptotic Analysis:

- Any useful insights can be obtained from the optimal solution?

Problem Decomposition

Primal Decomposition

- Define auxiliary variables:

$$\overline{P}_k = \boldsymbol{P}_k \cdot \Pi_k(\mathcal{P}),$$
$$\mathcal{P}_{main} = \{\overline{P}_1, \overline{P}_2, ...\overline{P}_K\}$$

(average transmit power allocated to user k)

- The optimization problem becomes:

Auxiliary variables

$$\overline{T}^* = \min_{\mathcal{P}_{main}, \mathcal{P}} \sum_{k=1}^{K} \frac{\overline{U}_k \tau}{\lambda_k} \qquad \overline{U}_k = \boldsymbol{Q} \cdot \Pi_k(\mathcal{P})$$

S.t: $p_k(\chi) \geq 0$

$$\pi_k(L) \leq \epsilon_d \quad \forall k \in \{1, 2...K\}$$

$$\sum_{k=1}^{K} \overline{P}_k \leq P_{avg}$$

Problem Decomposition

Primal Decomposition

For given $\mathcal{P}_{main}$, $\overline{U}_k$ is a function of $\mathcal{P}_k$ only and hence, we have:

$$\min_{\mathcal{P}_{main},\mathcal{P}} \sum_{k=1}^{K} \frac{\overline{U}_k \tau}{\lambda_k} = \min_{\mathcal{P}_{main}} \sum_{k=1}^{K} \min_{\mathcal{P}_k} \frac{\overline{U}_k \tau}{\lambda_k}$$

As a result, we can decompose the problem into one master problem + K subproblems

Problem 1 (Master Problem):

$$\overline{T}^* = \min_{\mathcal{P}_{main}} \sum_{k=1}^{K} \frac{\overline{U}_k^*(\overline{P}_k)\tau}{\lambda_k} \quad (18)$$

$$\text{S.t.:} \quad \sum_{k=1}^{K} \overline{P}_k \leq P_{avg} \quad (19)$$

Average Power allocation to the K users

Problem Decomposition

Primal Decomposition

Problem 2 (Sub Problem):

$$\overline{U}_k^*(\overline{P}_k) = \min_{\mathcal{P}_k} \boldsymbol{Q}\cdot\Pi_k(\mathcal{P}) \tag{20}$$

$$\text{S.t.:} \quad p_k(\chi_k) \geq 0 \tag{21}$$

$$\pi_k(L) \leq \epsilon_d \tag{22}$$

$$\boldsymbol{P}_k\cdot\Pi_k(\mathcal{P}) = \overline{P}_k \tag{23}$$

Instantaneous power allocation to the k-th user (subject to k-th user average power constraint $\overline{P}_k$)

Note that given ... lves according to it's own local ... and QSI only. Hence, we could write ...

Solution of the Subproblem

- **Transformation of Variables**
  - The subproblem is not convex w.r.t. the optimization variables $\{p_k(\chi_k)\}$
  - Using birth death dynamics of the problem, the subproblem is equivalent to:

$$\overline{U}_k^* = \min_{\overline{\mathcal{P}}_{k,Q}} \frac{\sum_{q=0}^{L} \frac{\prod_{i=q+1}^{L} \overline{\mu}_{k,i}^*(\overline{P}_{k,i})}{\lambda_k^{N-q}} q}{\sum_{q=0}^{L} \frac{\prod_{i=q+1}^{L} \overline{\mu}_{k,i}^*(\overline{P}_{k,i})}{\lambda_k^{N-q}}} \quad (27)$$

$$\text{S.t.:} \quad \frac{1}{\sum_{q=0}^{L} \frac{\prod_{i=q+1}^{L} \overline{\mu}_{k,i}(\overline{P}_{k,i})}{\lambda_k^{N-q}}} \leq \epsilon_d \quad (28)$$

$$\boldsymbol{\overline{P}} \cdot \Pi(\mathcal{P}_k) = \frac{\sum_{q=0}^{L} \frac{\prod_{i=q+1}^{L} \overline{\mu}_{k,i}^*(\overline{P}_{k,i})}{\lambda_k^{N-q}} \overline{P}_{k,q}}{\sum_{q=0}^{L} \frac{\prod_{i=q+1}^{L} \overline{\mu}_{k,i}^*(\overline{P}_{k,i})}{\lambda_k^{N-q}}} \leq \overline{P}_k \quad (29)$$

$$\overline{\mu}_{k,q}^*(\overline{P}_{k,q}) = \max_{\mathcal{P}_{k,q}} \mathbb{E}_{\mathbf{H}}[\mu_{k,q}(\chi) | Q_{k,m} = q] \qquad \overline{P}_{k,q} = \mathbb{E}[p_k(\chi_x) | Q_k = q]$$

Solution of the Subproblem

- **Transformation of Variables**
  - **Consider the following transformation:** $v_{k,q} = \prod_{i=q+1}^{L} \frac{\overline{\mu}^*_{k,i}(\overline{P}_{k,i})}{\lambda_k}, \ q \in \{0, 1, ...L\}$

(One-to-one mapping)

$$\mathcal{V}_k = \{\mu_{k,0}, ..., \mu_{k,L}\} \leftrightarrow \overline{\mathcal{P}}_k = \{\overline{P}_{k,0}, .., \overline{P}_{k,L}\}$$

  - **Transforming from the P domain to the V domain, the subproblem is equivalent to:**

$$\overline{U}^*_k = \min_{\mathcal{V}_k} \frac{\sum_{q=1}^{L} q v_{k,q}}{\sum_{q=0}^{L} v_{k,q}}$$

S.t.
$$\frac{1}{\sum_{q=0}^{L} v_{k,q}} \leq \epsilon_d$$
$$\frac{\sum_{q=1}^{L} F(\frac{v_{k,q-1}\lambda_k}{v_{k,q}}) v_{k,q}}{\sum_{q=0}^{L} v_{k,q}} \leq \overline{P}_k$$

$\Rightarrow$

$$\min_{\overline{U}_k, \mathcal{V}_k} \overline{U}_k$$

S.t.
$$\sum_{q=1}^{L} q v_{k,q} - \overline{U}_k \sum_{q=0}^{L} v_{k,q} \leq 0$$
$$1 - \epsilon_d \sum_{q=0}^{L} v_{k,q} \leq 0, \quad \sum_{q=1}^{L} F(\frac{v_{k,q-1}\lambda_k}{v_{k,q}}) v_{k,q} - \overline{P}_k \sum_{q=0}^{L} v_{k,q} \leq 0$$

Solution of the Subproblem

- **Global Optimal Solution**
  - **Theorem (Unique optimal solution): The subproblem has a unique global optimal solution. Furthermore, the following algorithm can reach the solution in $\lceil \log_2(\frac{L}{\varepsilon}) \rceil$ steps.**

Algorithm 1 (Bisection Searching):

- **Initialize:** Set $\overline{U}_{\min} = 0$; $\overline{U}_{\max} = L$.
- **Repeat:**
  - Set $\overline{U}_k = \frac{\overline{U}_{\min} + \overline{U}_{\max}}{2}$
  - Solve Problem 4 (defined below) using Algorithm 2;
  - **if** the optimal solution of Problem 4 $S_{\min} \leq 0$, $\overline{U}_{\min} = \overline{U}_k$, **else** $\overline{U}_{\max} = \overline{U}_k$;
- **Until** $\overline{U}_{\max} - \overline{U}_{\min} < \varepsilon$ where ε is the performance error tolerance bound. $\overline{U}_k^* = \overline{U}_k$.

Solution of the Subproblem

Structure of the Optimal Solution

- **Multi-level Power Water-filling:**

Water-level adaptive to QSI

Power Allocation according to water-filling w.r.t. CSI of users

$$p_k^*(\chi_k) = \left[\frac{1}{\alpha_{k,q}^* N_k} - \frac{1}{\mathbf{h}_k(\mathbf{w}_k\mathbf{w}_k^H)\mathbf{h}_k^H}\right]^+$$

- The water levels $\{\alpha_{k,q}^*\}$ can be determined offline based on long-term statistical information of the data source and CSI.

- Memory requirement is $\mathcal{O}(L)$

Solution of the Master Problem

- Recall that the master problem is to determine the "average power allocation" to the SDMA users $\mathcal{P}_{main} = \{\overline{P}_1, ..., \overline{P}_K\}$
- $\overline{U}_k^*(\overline{P}_k)$ is a convex function of $\overline{P}_k$ ➔ The master problem is convex in $\{\overline{P}_1, ..., \overline{P}_K\}$
- Form the Lagrangian function for the master problem
- Henc

Lemma 4.3 (Derivative of optimal buffer length w.r.t power constraint): Denote the lagrange multipliers corresponding to the optimal scheme $\mathcal{V}_k = \{v_{k,q}^*\}$ of Problem 4 when $\overline{U}_k = \overline{U}_k^*$ as $\beta_{k1}^*, \beta_{k2}^*$. The derivative of $\overline{U}_k^*(\overline{P}_k)$ w.r.t. $\overline{P}_k$ in the sub problem is given by (42). Moreover, $\frac{\partial \overline{U}_k^*}{\partial \overline{P}_k}$ is a non-decreasing function of $\overline{P}_k$.

$$\frac{\partial \overline{U}_k^*}{\partial \overline{P}_k} = -\frac{\beta_{k2}^*}{1 - \beta_{k1}^* - \beta_{k2}^*} \tag{42}$$

- How to detemine the subgradient?

Asymptotic Analysis

Structure of the Water-Levels

Lemma 5.2 (Asymptotic closed-form expression of $\{\alpha^*_{k,q}\}$ in terms of $\alpha_{k,1}$): The water-filling levels under different QSIs is an geometric series:

$$\frac{1}{\alpha_{k,q}\overline{N}_k} = \mathcal{O}\left(\left(\frac{\log(\frac{1}{\alpha^*_{k,1}\overline{N}})}{\lambda_k \overline{N}_k}\right)^{q-1} \frac{1}{\alpha_{k,1}\overline{N}_k}\right), \; q \in \{1, 2, ...L\}. \qquad (45)$$

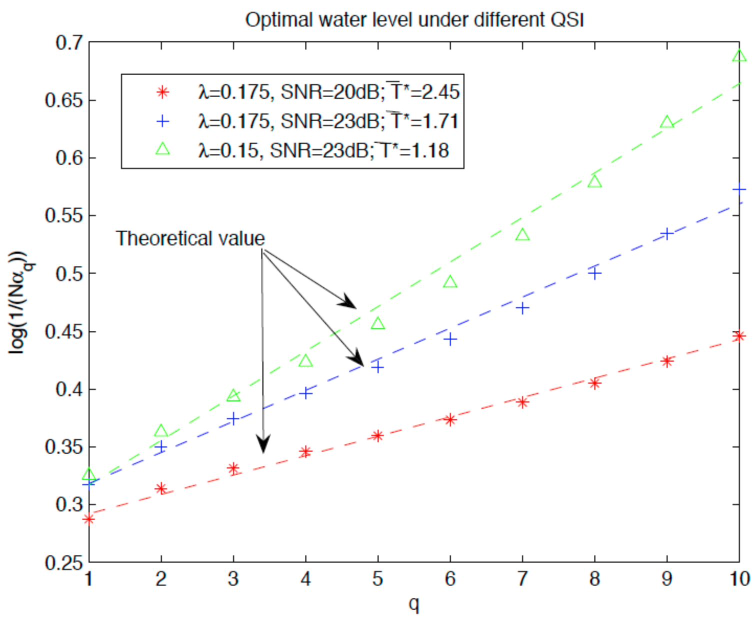


Fig. 3. Relationship of water levels in the proposed multi-level water-filling solution. The y-axis is log of water level and the x-axis is the QSI. We assume L = 10 and $SNR = 10\log 10(\overline{P}_k)$

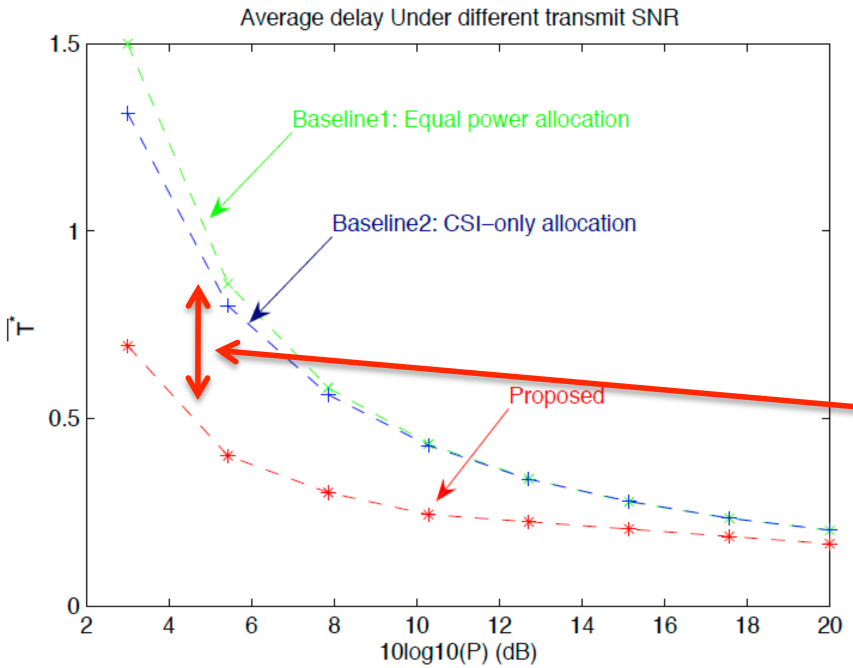
$\log\left(\frac{1}{\alpha^*_{k,q}}\right)$ forms an arithmetic series

$\rightarrow \{\alpha^*_{k,q}\}$ forms a geometric series

Asymptotic Analysis

Corollary 5.1 (Performance gain compared to the CSI-only policy): Optimal buffer length $\overline{U}_k^*$ achieved by the proposed multilevel water-filling algorithm is $\frac{\lambda_k}{\mathcal{O}(\log \overline{P}_k)+\mathcal{O}(\log\log \overline{P}_k)-\lambda_k}$ while that achieved by the traditional CSI-only (single-level water-filling) policy is $\frac{\lambda_k}{\mathcal{O}(\log \overline{P}_k)-\lambda_k}$.

Gain due to multi-level water-filling



Significant delay gain compared with CSI-only scheme

Fig. 6. Average delay versus average SNR. The baseline 1 and 2 scheme correspond to equal power scheme and *CSI-only* scheme (single-level water-filling) respectively. $nT = 5$, $K = 4$, $\lambda_k = 0.09 + 0.01 * k$, $k \in \{1, ...K\}$, Maximum buffer $L = 10$ and maximum packet drop rate $\epsilon_d = 0.01$, $SNR = \frac{10 \log 10(\overline{P})}{K}$

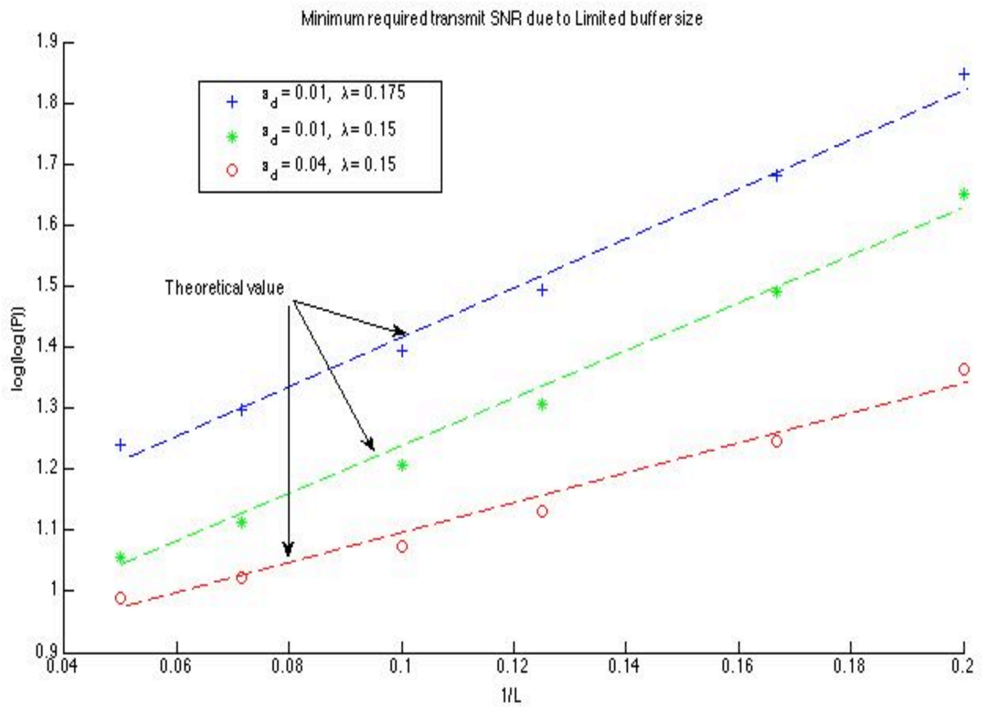
Asymptotic Analysis

Buffer Length Requirement

Corollary 5.2 (Minimum power required due to finite buffer size): Denote $\overline{P}_{k,\min}$ as the minimum power to achieve the packet drop rate constraint ϵ_d under a maximum buffer size L.

$$\log\log(\overline{P}_{k,\min}) \propto \frac{-\log\epsilon_d}{L} + \log(\lambda_k) + \log(\overline{N}) \quad (48)$$

First order guideline on buffer dimensioning



For small ε_d

$$[\log\log SNR_{min}] \times L = \text{constant}$$

Conclusion

Conclusion 1 (Structure of Delay-Optimal Power Control):

Delay-Optimal Power Allocation – multilevel water-filling: Water-filling across CSI, water level determined by QSI.

Conclusion 2 (Complexity):

Low complexity O(K) solution via stochastic decomposition and birth-death queue dynamics

Conclusion 3 (Asymptotic Results):

Gain of multilevel water-filling is loglog SNR.

Buffer Length x log log SNR = constant

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Thank you!

Questions are Welcomed!

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